K25P 2895

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (C.B.C.S.S. – OBE – Reg./Supple./Imp.) Examination, October 2025 (2023 Admission Onwards) MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND MATHEMATICAL ANALYSIS, MODELLING AND SIMULATION, FINANCIAL RISK MANAGEMENT) MSMAT03C13/MSMAF03C13 : Differential Geometry

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 5** questions from the following 6 questions. **Each** question carries **4** marks.

1. Define the Gradient Vector field. Find the gradient vector field of the function $f(x_1, x_2) = x_1 - 3x_2^2,\ x_1, x_2 \in \mathbb{R}$.

2. Sketch the graph of the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x_1, x_2) = x_1^2 - x_2^2$.

3. Prove or disprove the following : the geodesics have constant speed.

4. Compute $\nabla_v f$ where $f(x_1, x_2) = 2x_1^2 + x_1 x_2^2$, v = (1, 0, 1, –2).

5. Define the following terms :
   i) Normal curvature of a surface ii) Principle curvature of a surface.

6. With usual notations, prove that d(f + g) = df + dg. **(5×4=20)**

PART – B

Answer **any 3** questions from the following **5** questions. **Each** question carries **7** marks.

7. Show that the unit n sphere is an n surface in $\mathbb{R}^{n+1}$.

8. Sketch the level set $f^{-1}(c)$ for n = 2 of the function $f(x_1, x_2, ..., x_{n+1}) = x_1^2 + x_2^2 + x_3^2 + ... - x_{n+1}^2$ at c = –1, 0, 1.

P.T.O.

9. Let S denote the cylinder $x_1^2 + x_2^2 = 1$ in $\mathbb{R}^3$. Show that α is a geodesic of S if and only if α is of the form $\alpha(t) = (\cos(at + b), \sin(at + b), ct + d)$ for some $a, b, c, d \in \mathbb{R}$.

10. Prove that the Weingarton map is self-adjoint.

11. Compute $\int_C (-x_2 dx_1 + x_1 dx_2)$, where C is the ellipse $(x_1^2/a^2) + (y^2/b^2) = 1$ oriented by its inward normal. **(3×7=21)**

PART – C

Answer **any 3** questions from the following 5 questions. **Each** question carries **13** marks.

12. a) Find the integral curve through (1, 0) of the vector field $X(x_1, x_2) = (x_1, x_2, x_2, x_1)$.
    b) State and prove the Lagrange Multiplier Theorem.

13. Prove the following : Let S be a compact connected oriented n–surface in $\mathbb{R}^{n+1}$ exhibited as a level set $f^{-1}(c)$ of a smooth function $f : \mathbb{R}^{n+1} \to \mathbb{R}$ with $\nabla f(p) \neq 0$ for all $p \in S$. Then the Gauss map maps S onto the unit sphere S^n.

14. a) Prove the following : Let S be an n surface in $\mathbb{R}^3$ and $\alpha : I \to S$ be a geodesic in S with $\dot{\alpha} \neq 0$. Then a vector field X tangent to S along α is parallel along α if and only if both $\|X\|$ and the angle between X and $\dot{\alpha}$ are constant along α.
    b) Show that in an n–plane, parallel transport is path independent.

15. a) Let $g : I \to \mathbb{R}$ be a smooth function and let C denote the graph of g. Show that the curvature of C at the point $(t, g(t))$ is $g''(t)/(1 + (g'(t))^2)^{3/2}$, for an appropriate choice of orientation.
    b) Prove the following : Let η be the 1–form on $\mathbb{R}^2 - \{0\}$ defined by $\eta = -\frac{x_2}{x_1^2 + x_2^2} dx_1 + \frac{x_1}{x_1^2 + x_2^2} dx_2$. Then for $\alpha : [a, b] \to \mathbb{R}^2 - \{0\}$ be any closed piece wise smooth parametrized curve in $\mathbb{R}^2 - \{0\}$, $\int_\alpha \eta = 2\pi k$.

16. a) Find the Gaussian curvature of the ellipsoid $x_1^2/a^2 + x_2^2/b^2 + x_3^2/c^2 = 1$ oriented by its inward normal.
    b) Let $\phi : U_1 \to U_2$ and $\psi : U_2 \to \mathbb{R}^k$ be smooth, where $U_1 \subset \mathbb{R}^n$ and $U_2 \subset \mathbb{R}^m$. Verify that $d(\Psi \circ \phi) = d\Psi \circ d\phi$. **(3×13=39)**
